Things you will need to know for the Real Analysis in MATH20101

Differentiation

In Questions 3-8 I give some of the examples of functions that have had to be differentiated in the previous few exams for this course.

3) (2003) Differentiate

i) $\ln(1+x) - x$, for $x > 1$, twice,

ii) $x^2 + \tan^2 x$, when $x \neq (2k+1)\pi/2$ for any $k \in \mathbb{Z}$.

iii) $x \tan x$, when $x \neq (2k+1)\pi/2$ for any $k \in \mathbb{Z}$.

iv) $e^{-x} \sin x$, 9 times.

4) (2004) Differentiate $\cos^2 x$, 8 times,

5) (2006) Differentiate

$$\frac{x}{\sin x} + \frac{1}{\cos x},$$

when $x \neq \pi\ell/2$ for any $\ell \in \mathbb{Z}$.

6) (2007) Differentiate

i) $\ln^2(1+x)$, for $x > -1$,

ii)

$$\ln\left(\frac{1+x}{1-x}\right),$$

for $x > 1$, 5 times.

iii) $\tan^2 x$, when $x \neq (2k+1)\pi/2$ for any $k \in \mathbb{Z}$.

iv) $e^x(\sin x + \cos x)$, 6 times,

7) (2008) Differentiate

i) $2^x - x^2$, 3 times.

ii)

$$\frac{e^x}{1+x},$$

for $x \neq -1$, 5 times.

Alternatively, let $f(x) = e^x/(1+x)$ so $(1+x)f(x) = e^x$ and differentiate this 5 times.

iii) $x \sinh x + \cosh x$, 6 times

8) (2009) Differentiate

i)

$$x^2 \sin \frac{\pi}{x^2}.$$

for $x \neq 0$.

ii) $\ln^2 (1 + x)$, for $x > -1$, 5 times,

Practice of differentiation.

On the basis that you learn mathematics only by doing mathematics here are a series of questions on differentiation to help you learn, or perhaps refresh, your facility on differentiating standard functions.

9) Use the **Product Rule** to differentiate

i) $e^{2x} = (e^x)^2$,

ii) $e^x \ln x$, for $x > 0$,

iii) $(\sin x)(\ln x)$, for $x > 0$,

iv) $e^x \cos x \ln x$, for $x > 0$,

10) Use the **Quotient Rule** to differentiate

i) $e^{-x} = 1/e^x$,

ii) e^x/x, for $x \neq 0$,

iii) x/e^x,

iv) $\ln x/x^2$, for $x > 0$,

v) $x^2/\ln x$, for $x > 1$,

vi) $1/\tan^2 x$, when $x \neq \pi\ell$ for any $\ell \in \mathbf{Z}$.

vii) $(\sin x + \cos x)/(\sin x - \cos x)$, when $x \neq (1 + 4k)\pi/4$ for any $k \in \mathbb{Z}$.

11) Use the **Chain Rule** to differentiate

i) e^{2x},

ii) e^{-x},

iii) $\cos(1/x)$, for $x \neq 0$,

iv) $\sin(\ln x)$, for $x > 0$,

v) $\ln^{1/2} x$, for $x > 1$,

vi) $\ln\left(1 + \cos\left(x^{1/2}\right)\right)$, for $x > 0$,

vii) $e^{\sqrt{1+\sin^2 x}}$,

12) Differentiate

i) $\sin(x \ln x)$, for $x > 0$.

ii) $\left(x^{1/2} \sin^2 x\right) / (x^2 + 1)$, for $x > 0$,

iii) $(\sin x/x)^2$ for $x \neq 0$,

iv) $x \sin\left(\frac{1}{x}\right)$, for $x \neq 0$,

v) $x^m \sin\left(\frac{1}{x^n}\right)$, $m \geq 1, n \geq 1$ and $x \neq 0$,

vi) $\sin^m x \cos^n x$, $m \geq 1, n \geq 1$,

vii) $\sqrt{2\sin^2 x + 4\cos^2 x}$,

viii) $x^m \ln^n x$, $m \geq 1, n \geq 1$ and $x > 0$,

ix) $e^{\sqrt{\ln x}}$ for $x > 1$,

x) $e^{1/\sqrt{\ln x}}$ for $x > 1$,

xi) $\ln\left(1 + \sqrt{1 + e^x}\right)$. Solution: $\frac{1}{2\sqrt{(1+e^x)}} \frac{e^x}{1+\sqrt{(1+e^x)}}$